

## BINARY PLANETESIMALS AND THEIR ROLE IN PLANET FORMATION

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## Abstract

One of the main evolutionary stages of planet formation is the dynamical evolution of planetesimal disks. These disks are thought to evolve through gravitational encounters and physical collisions between single planetesimals. In recent years, many *binary* planetesimals have been observed in the Solar system, indicating that the binarity of planetesimals is high. However, current studies of planetesimal disks formation and evolution do not account for the role of binaries. Here we point out that gravitational encounters of binary planetesimals can have an important role in the evolution of planetesimal disks. Binary planetesimals catalyze close encounters between planetesimals, and can strongly enhance their collision rate. Binaries may also serve as additional heating source of the planetesimal disk, through the exchange of the binaries gravitational potential energy into the kinetic energy of planetesimals in the disk.

*Subject headings:* planets and satellites: formation — protoplanetary disks — planets and satellites: dynamical evolution and stability — minor planets, asteroids: general

## 1. INTRODUCTION

Current models for the evolution of planetesimal disks consider three basic processes: viscous stirring, dynamical friction and coagulation or disruption through collisions (e.g. Safronov 1972; Lissauer 1993; Goldreich et al. 2004). These processes account for the rate at which single planetesimals encounter one another and collide, thereby changing the sizes (and masses) of planetesimals and their velocity distribution. None of these processes, however, takes into account the role of binary planetesimals.

An important question when discussing the role of binaries is whether they exist at all in planetesimal disks during their evolution. Evidently, in the Solar system many binary planetesimals (BPs) formed and survived to this day (see Goldreich et al. 2002; Weidenschilling 2002; Durda et al. 2004; Funato et al. 2004; Richardson & Walsh 2006; Lee et al. 2007; Schlichting & Sari 2008; Perets & Naoz 2009; Nesvorný et al. 2010, for possible formation and evolutionary scenarios). Large binary fractions ( $> 10\%$ ) are currently observed both for asteroids and trans-Neptunian objects (TNOs) in the Solar system (Noll et al. 2008; Kern & Elliot 2006; Mann et al. 2007; Richardson & Walsh 2006; Walsh 2009). These are found to have separations of up to tens or hundreds times the radii of their single planetesimal components (for asteroids and TNOs, respectively; up to 0.1 the Hill radii for these objects). The currently observed large fractions of BPs in the Solar-system therefore serves as a basic motivation for our study. We assume that such BPs population exists in a given planetesimal disk, and study its role in the planetesimals evolution. We note that a variety of mechanisms were suggested for the origin of BPs in the Solar system (Richardson & Walsh 2006; Nesvorný et al. 2010), but only some limited aspects of their evolution have been explored (e.g. Perets & Naoz 2009 and references therein).

The role of binaries in gravitational systems have been studied in depth in the context of stellar clusters (Hills 1975; Spitzer 1987; Hut et al. 1992). In such systems stellar binaries are known to serve as important dynamical heating source of the clusters (so called binary heating), which slows the collapse of the cluster. Binary stars are also known to have a major role in introducing stellar collisions and accelerate the growth of massive stars; it is thought that most stellar collisions are the result of encounters between binary and single (or other binary) stars. Here we suggest that BPs can play a similar role in planetesimal disks. The cross section for close encounters of binaries can be much larger than that of single planetesimals, thereby increasing their collision rates. The potential energy of close binaries could be exchanged into kinetic energy of the planetesimals (and vice-versa), and therefore serve as an additional heating (cooling) source for the planetesimal disk. In the following we explore both the process of binary induced collisions as well as binary heating.

## 2. BINARY ENCOUNTERS

The gravitational interactions between binary and single planetesimals can lead to various outcomes including binary disruptions, exchanges, resonant encounters and physical collisions.

In binary-single encounters energy is exchanged between the internal orbital energy of the binary,  $E_{bin}$ , and the kinetic energy of the incoming perturber,  $E_k$ . These are given by  $E_{bin} = Gm_1m_2/2a$ , where  $m_1, m_2$  are the binary-components masses and  $a$  is the binary mutual separation, and  $E_k = 0.5mv^2$ , where  $m$  is the typical mass of the perturbing single planetesimals, and  $v$  is the relative velocity between the planetesimals (typically of the order of the velocity dispersion of the planetesimals). A binary is termed soft if  $|E_{bin}|/E_k < 1$  and hard if  $|E_{bin}|/E_k > 1$ . On average, hard binaries get harder following an encounter, and soft binaries get softer (the so called Heggie's law; Heggie 1975). Soft binaries rapidly evaporate through such encounters and have a weak af-

fect on the energy budget of the system. However, encounters with hard binaries lead, on average, to the significant loss of orbital energy from the binary, making the binary harder, following the scatter of the perturbing object into higher velocity. Therefore such a process leads to the increase of the planetesimals velocities and the dynamical heating of the system. Alternatively, a physical collision may occur between two or all of the planetesimals involved in the encounter. In the following we discuss the role of both these possibilities.

### 2.1. Binary induced collisions

The collision rate of a planetesimals is given by

$$\Gamma = n\sigma v, \quad (1)$$

where  $n$  is the number density of planetesimals in the disk,  $\sigma$  is the cross section for the collision and  $v$  is the relative velocity between the planetesimals. The cross section for a physical collision depends on the relative velocity between particles. When the relative velocity between planetesimals is much larger than the escape velocity of the most massive planetesimal participating in the encounter, gravitational focusing is negligible and the cross section is given by the projected cross sectional of the planetesimal  $\sigma \sim \pi r^2$  ( $r$  is the planetesimal radius). When the relative velocity is slower, gravitational focusing becomes dominant and we obtain the following cross section ( $\sigma_1$ ) for two single planetesimal collisions

$$\sigma_1 \approx \pi r^2 \left(1 + \frac{v_e}{v}\right)^2 \sim \pi r^2 \left(\frac{v_e}{v}\right)^2, \quad (2)$$

where  $v$  is the relative velocity and  $v_e$  is the escape velocity from the planetesimal surface (with the right most term obtained for  $v_e \gg v$ ).

Binary-single encounters could involve complex trajectories, as the encounter now involves three bodies and the planetesimal trajectories can become chaotic (binary-binary encounters, not discussed here, interact in even more complex way). In such encounters the probability for a direct collision between any two (or even all) of the objects involved is highly increased (Hills & Fullerton 1980; Fregeau et al. 2004), and therefore binaries serve as efficient catalysts for direct physical collisions. The cross section for a physical collision during a single-binary encounters,  $\sigma_2$ , is approximately given by Fregeau et al. (2004; also Fregeau, private communication and (Sigurdsson & Phinney 1993; Valtonen & Karttunen 2006) and references therein)

$$\sigma_2 \approx \pi a^2 \left(\frac{v_c}{v}\right)^2 \left(\frac{r/10 \text{ km}}{a/2.14 \times 10^3 \text{ km}}\right)^{0.65}, \quad (3)$$

where  $v_c$  is the critical velocity separating soft and hard binaries and  $a$  is the binary semi-major axis. Note that this equation was derived for stellar encounters, however, the dynamics of gravitational encounters is scale free, and can be scaled for the use of planetesimals mass objects. Although physical collisions also depend on the density of the objects, the typical average density of planetesimals ( $0.5 - 3 \text{ gr cm}^{-3}$ ) is comparable to that of stars, and therefore scaling of mass and radii could be used.

For simplicity we will consider equal mass planetesimals. For this case  $v_c$  is approximately the mutual orbital velocity of the binary components (similar to the

escape velocity from the binary at its separation,  $a$ ; i.e.  $v_c \propto v_e(r/a)^{1/2}$ ) and we therefore get

$$\frac{\sigma_2}{\sigma_1} \approx 33 \left(\frac{a}{r}\right)^{0.35}. \quad (4)$$

As can be seen from this ratio, collisions during binary-single encounters could be tens up to hundreds of times more frequent than single-single encounters for typical BPs currently observed in the solar system<sup>1</sup>. The ratio between collisions rates due to binary-single encounters ( $\Gamma_2$ ), vs. single-single encounters ( $\Gamma_1$ ) is therefore given by

$$\frac{\Gamma_2}{\Gamma_1} \approx \frac{n_2 \sigma_2 v}{n_1 \sigma_1 v} \approx 39 \left(\frac{n_2/n_1}{0.3}\right) \left(\frac{a/r}{50}\right)^{0.35}, \quad (5)$$

where we took a binary fraction of  $f_{bin} = n_2/n_1 = 0.3$  in the normalization.

Consequently, the rate of physical collisions and the mass growth of planetesimals (which is proportional to the collision rate,  $\dot{M} \propto \Gamma$ ) could be boosted and dominated by binary-single encounters even for a low binary fraction. Moreover, binary-single encounters produce a different (power law) dependence of the growth rate on the planetesimals size. They also introduce dependencies on the *binary* parameters, namely the binaries separation and binary fraction (which could be size dependent by themselves). The mass growth rate of planetesimals could therefore change both quantitatively (faster) and qualitatively due to existence of BPs. This could affect the size distribution of planetesimals in protoplanetary disks (or debris disks) and its evolution. Hence, the orbital properties of binaries (Naoz et al. 2010), the size distribution of planetesimals, and their coupling could all be used, in principle, to characterize and constrain the evolution of the Solar system.

Taking a conservative binary fraction of 10 percents for binary asteroids with typical separations of 10 times the planetesimal radius (e.g. typical of main belt asteroids; Richardson & Walsh 2006), we get  $\Gamma_2/\Gamma_1 \sim 7$ . The binary fraction of TNO binaries could be higher than 30 percents with typical separations of  $a \sim 50 r$  (e.g. typical for binary TNOs; Richardson & Walsh 2006), leading to collision rate as high as  $\sim 39$  times higher than the expected single-single physical collision rate. Given the likely higher binary fraction of planetesimals, both today and in the past, collision rates catalyzed by BPs during planet formation were likely to be even higher. Nevertheless, more detailed study of the formation and destruction mechanisms of BPs is required to asses this question quantitatively.

We note that in the case of single-single collisions, the planetesimals are unbound prior to collision, whereas in the binary-single case the colliding planetesimals could have been marginally bound (e.g. during resonant encounters). The expected impact velocities during binary-single encounters are therefore likely to be lower, on average, than those expected in single-single collisions. Lower

<sup>1</sup> Note that this ratio exceeds unity even as we approach to  $r = a$ ; in these cases the flyby of a perturber can easily perturb the binary components into collision, even if the perturber never crosses between the binary components.

velocity collisions are more likely to result in mass accretion rather than shattering of planetesimals (the likely outcome of higher velocity impacts), and are therefore more efficient for planetesimals growth. The accelerated collision rate may also imply that more collisions occur at earlier times, when the planetesimal disk is cooler, again leading to typically lower velocity impacts.

## 2.2. Binary heating

In the following we focus on the more energetically important hard-binaries. Encounters with hard binaries lead, on average, to the loss of orbital energy from the binary and the scattering of the perturbing object into higher velocity. Therefore such a process leads to the heating of the system (Heggie 1975; Hills 1975; Spitzer 1987). In fact, in stellar clusters, this “binary heating” process is considered as one of the main processes governing the evolution of the clusters (Hut et al. 1992). We can now consider the effect of binary heating on the evolution of a planetesimal disk. We first note, however, that even the initial formation of binaries would heat the planetesimals disk (or the gas if the binaries formed due to gas interactions), since the binaries binding energy have had to be transferred to the disk planetesimals upon their formation.

### 2.2.1. Energy budget

One can estimate the amount of potential energy reservoir available for hard binaries. A binary can be hardened up to the point when it becomes a contact binary. The potential energy of such binary is of the order of

$$E_{bin} \approx \frac{Gm^2}{2r} \quad (6)$$

where  $r$  is the typical radius of a planetesimal of mass  $m$ . The energy extracted from a binary, which was initially at some typical separation  $a \gg r$  is of the order of

$$\Delta E_{bin} = E_{fin} - E_{init} = \frac{Gm^2}{2} \left( \frac{1}{r} - \frac{1}{a} \right) \approx \frac{Gm^2}{2r} = E_{bin}. \quad (7)$$

If all this energy were to be gained by the planetesimals in the disk, they should have been excited to higher velocity dispersion, with

$$\Delta E_{kin} = N \frac{m \Delta v^2}{2} \approx N_{bin} \frac{Gm^2}{2r} = N_{bin} E_{bin}, \quad (8)$$

where  $\Delta v$  is the change in the velocity dispersion of the planetesimals, and  $N$  and  $N_{bin}$  are the number of single and binary planetesimals, respectively. We therefore find that the change in velocity is

$$\Delta v \approx \sqrt{G \frac{N_{bin} m}{N r}} = \sqrt{f_{bin} \frac{Gm}{r}}, \quad (9)$$

where  $f_{bin}$  is the binary fraction. Comparing this to the Hill velocity,  $v_H = (Gm/R_H)^{1/2}$  (where  $R_H \simeq A(3m/M_\odot)^{1/3}$  is the Hill radius at distance  $A$  from the Sun), we find that

$$\Delta v \sim \sqrt{f_{bin} \frac{R_H}{r}} v_H. \quad (10)$$

Typically, the radii of planetesimals are much smaller than the Hill radius by a few orders of magnitude. Therefore, even a small binary fraction potentially holds enough energy to heat up a disk to Hill and even super-Hill velocities. This could have important implications on the binary formation mechanism involved. For example, binary formation through dynamical friction discussed by Goldreich et al. (2002), is efficient only for relatively low velocity dispersions of the small planetesimals. This may imply that the high binary frequency observed in the Solar system is limited to large planetesimals; e.g. the energy extracted from the binaries formation is distributed over a much larger mass of low mass planetesimals, which never formed binaries by themselves as these were all too soft to survive and contribute to the binary heating. Alternatively, binary formation could have been primordial, when they were embedded in gas, or during the collapse of planetesimals swarms forming single planetesimals (Nesvorný et al. 2010). In these latter cases the extra energy from the binaries formation would be dissipated in the gas or radiated as heat during the gravitational collapse. Better understanding of binaries formation is required for resolving this issue.

### 2.2.2. Heating rate

We now turn to the rate at which binaries heat the system. For simplicity we will consider a planetesimal disk composed of two types of objects: single planetesimals of mass  $m$ , number density  $n$ , and typical velocity dispersion  $v$  and hard binaries of mass  $m_{bin}$  (where we assume the mass ratio,  $q$ , is typically high), number density  $n_{bin}$ . The rate of energy gain by the system from a single hard binary is then given by (Hills & Fullerton 1980; Spitzer 1987; Hills 1992)

$$\frac{dE_{bin}}{dt} = D_2 G^2 \frac{n m m_{bin}^2}{v_{enc}}, \quad (11)$$

where  $n$  is the number density of planetesimals,  $v_{enc}$  is the typical encounter velocity ( $\sim$  the velocity dispersion of the planetesimals), and  $D_2$  is some constant pre-factor, which will be discussed later on. We now use the relation  $dE/dt = 3mvdv/dt$  (the 3 factor comes from considering the velocity dispersion in a 3D system) to find expressions for the binary heating of the single planetesimals. Taking

$$n \frac{dE}{dt} = -n_{bin} \frac{dE_{bin}}{dt} \quad (12)$$

and translating this into evolution of the velocity dispersion (i.e. the energy extracted from the binaries is the kinetic energy gained by the planetesimals), we find

$$\left( \frac{dv}{dt} \right)_{bin} = \frac{n_{bin}}{n} \frac{1}{3mv} \frac{dE_{bin}}{dt} = D_2 G^2 \frac{n_{bin} m_{bin}^2}{3v v_{enc}} \approx D_2 G^2 \frac{n_{bin} m_{bin}^2}{3v^2}, \quad (13)$$

where numerical simulations (Hills 1992) show that the constant pre-factor,  $D_2$ , is of the order 6.7 for encounters of single mass objects (i.e.  $q \sim 1$  and  $m_{bin} = 2m$ ) and about twice as large for the case of more massive binaries ( $m \ll m_{bin}$ ). In the last term we assume the encounter velocity of the same order of the velocity dispersion of the planetesimals.

We can compare the importance of binary heating with that of viscous heating by single planetesimals. Let us use a simple estimate for the viscous heating, following a similar approach by Alexander et al. (2007), taking a simplifying assumption that the velocity dispersion is isotropic. This is not strictly valid, but it has been shown that the ratio of the radial and vertical velocity dispersions cannot become larger than 3 without the system becoming unstable (Kulsrud et al. 1971; Poliachenko & Shukhman 1977). The relaxation time for such a system is given by

$$t_{\text{relax}} = \frac{v^3}{CG^2nm^2\ln\Lambda}, \quad (14)$$

(e.g. Binney & Tremaine 1987; Papaloizou & Terquem 2001 where  $n$  is the planetesimals density,  $v$  is the typical velocity dispersion of the planetesimals and  $\ln\Lambda$  is the Coulomb logarithm (where  $\Lambda \sim H/r$ ;  $H$  is the disk scale height) and  $C$  is an order-of-unity constant that depends on the geometry of the system. (for a spherical system  $C \simeq 2.94$ , Binney & Tremaine 1987). Consequently, the relaxation of the system is governed by

$$\left(\frac{dv}{dt}\right)_1 = D_1 G^2 \frac{nm^2 \ln\Lambda}{3v^2}, \quad (15)$$

with  $D_1 = 2C$ . Comparing with the binary heating term in Eq. 13 we find both heating mechanism have similar dependencies on the disk and planetesimals properties. Taking the same velocity dispersion for both binaries and single planetesimals, and considering the same mass for all planetesimals ( $m_{\text{bin}} = 2m$ ), the two heating terms differ only by some constant pre-factor, and the relative binary fraction, i.e.

$$\left(\frac{dv}{dt}\right)_{\text{bin}} / \left(\frac{dv}{dt}\right)_1 = \frac{4D_2}{D_1} \frac{n_{\text{bin}}}{n} = \frac{4D_2}{D_1} f_{\text{bin}}. \quad (16)$$

Taken at face value  $4D_2/D_1 \sim 4.5$ , and therefore the binary heating contribution could be comparable to that of viscous heating and dynamical friction, for realistic binary fractions of 0.1 – 0.2. Note that in the case of dynamical friction effect on low mass planetesimals, we should take the binary mass to be larger than the mass of the heated planetesimals and  $D_2$  would be twice as large (Hills 1992). Given these uncertainties and the simplifications used in the above derivations, one should be cautious in using them at face value. These findings do, however, suggest that binary heating is likely not negligible, but is also not likely to be the single most dominant mechanism for heating of the planetesimals disk.

Binary heating would continue as long as hard binaries exist in the disk. However, as with the other forms of planetesimal disk heating, heating the disk puffs it up, resulting in a larger volume and hence lower number density of planetesimals and lower encounter rates.

In addition, as the velocity dispersion is increased, more binaries become soft, and could be disrupted. We note that all of the suggested binary origin scenarios become less efficient at higher velocity dispersions, as the number density of planetesimals decrease and gravitational focusing becomes less effective. Currently, most of the observed BPs are soft in terms of encounters with similar size/mass planetesimals, and none of the binary

planetesimals formation scenarios is currently effective in producing new binaries (Richardson & Walsh 2006; beside the formation of low mass binaries through radiative spin up in the inner regions of the Solar system). Therefore binary formation and major collisional evolution had to proceed at early times, and the observed binary fraction today is representative only of the survived binaries, where as the earlier binary fraction was likely to be higher.

### 3. CAVEATS

**Stellar vs. planetesimal encounters:** The main caveat in our discussion of the role of BPs is the use of the stellar encounters approximation for estimating planetesimals encounters. Such approximations neglects the effect of the Sun on the encounters.

The important regime for binary-single encounters is when the impact parameter of the encounter is smaller than the binary separation (and the Hill radius of the binary). Such encounters therefore occur mainly in the regime dominated by the mass of the planetesimals rather than the Sun. However, during resonant encounters, planetesimals can be scattered to distances larger than the Hill radius, and still come back to re-encounter the binary in the absence of the Sun potential, at which point the tidal forces induced by the Sun could perturb them. Most resonant orbits, however, are likely to be at smaller separations (see e.g. Hills 1983, for a related, although different problem); in fact the probability of being ejected to some large distance  $r \gg a$  is comparable to the probability of the binary becoming unbound altogether (see Valtonen & Karttunen 2006, chapter 8.3). Such perturbation and possible quenching of the interaction due to the Sun would therefore mostly affect wide binaries, with separations close to their Hill radius. Note, however, that when taking into account the gravitational pull of the Sun, the interaction even between single planetesimals could become resonant, basically leading to the temporary capture of the planetesimals and a long lived interaction (Astakhov et al. 2005; Rafikov & Slepian 2010). The additional interaction with the Sun can therefore increase the rate of resonant encounters in which the the BPs interact chaotically and therefore have a higher probability for physical collisions. Few body simulation of such binary-single planetesimals, which are beyond the scope of this letter could give a more quantitative picture of these processes.

**Collisional cooling:** In the discussion of binary heating we neglected the effect of physical collisions. The amount of energy damping through collisions is not very well understood for planetesimals and depend on the (unknown) coefficient of restitution of the planetesimals. The high collision rate during encounters could therefore serve an important role in the heating/cooling rate by binaries. We note that the importance of collisions is poorly known even in the much better studied literature of binaries in stellar clusters, where it was suggested to play an important role mostly for the hardest (tidally captured) binaries (McMillan 1986) making binary heating much less efficient.

### 4. DISCUSSION AND SUMMARY

In this *letter* we pointed out the importance of BPs and their role in the evolution of protoplanetary and debris

disks. They could affect the evolution of such disks both through efficiently catalyzing physical collisions between planetesimals, and by serving as an additional planetesimal heating/cooling mechanism.

Binary encounters provide a novel and efficient mechanism for planetesimals growth, suggesting a different dependence between the growth rate of planetesimals and their physical size. They also introduce dependencies on the binary properties, such as the binary fraction and semi-major axis (and their size dependent distribution). The evolution of the planetesimals size distribution in this case could therefore be qualitatively different than that envisioned in studies taking into account only single planetesimals (e.g. Dohnanyi 1969; Davis & Farinella 1997; Kenyon & Bromley 2004, and other related papers and references therein)].

The relative importance of binaries depends on the binary fraction of planetesimals. Current observations of the Solar system show large BPs (asteroids; TNOs) populations, even amongst the largest planetesimals/embryos, e.g. Pluto-Charon and the Earth-moon systems. BPs therefore exist even at late stages in the planetesimal disk, even up to the the scale of planetary embryos. BPs may therefore accelerate the mass growth of planetesimals both at early stages of evolution (when they were

suggested to form; Goldreich et al. 2002), and possibly even up to the stages relevant for the formation of the gas planets cores.

We note that approaches used for the study of binaries in stellar clusters, could similarly be used to confront the new challenges raised by accounting for the role BPs, but we caution, and raise some caveats for their direct use in this context.

Current studies of planet formation do not take into account BPs. Such additional component could be difficult to account for in these studies (especially in simulations where following binary orbits is computationally expensive). Nevertheless, our findings suggest that not only that binaries are not negligible, but they may have an important role in the evolution of planetesimal disks, and therefore should not be ignored. We conclude that the inclusion of binaries in future studies could have important implications for our understating of the evolution of planetesimal disks and planet formation.

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